Mechanics of brittle fracture

Course contents :

The class presents the basics of linear elastic fracture mechanics. Failure of structures is explained by studying the physic of materials. This behavior is then modeled analytically. Finally numerical methods applied in fracture mechanics are presented. The course “Mechanics of Brittle Fracture” is proposed for students who want to begin to understand, apply and contribute to this important field of Solid Mechanics. It is assumed that the student is familiar with the theory of linear elasticity, vector calculus, linear algebra and indicial notation.

Subjects covered by the lectures are

- Some Fundamental Aspects of Structural Design and Failure Analysis
- Linear elastic fracture mechanics (LEFM)
- Energy Flows in Elastic Fracture
- Criteria for Elastic Fracture
- Determining K and G
- Linear Elastic Stress Analysis of 2D Cracks (Modes of Fracture)
- Analytical solutions of linear fracture mechanics boundary value problems (Complex Variables Method for Stress Analysis of Cracks)
- The Cohesive Zone Model Approach
- Three Dimensional Cracks
- Fatigue (Low Cyclic Fatigue, Fatigue Crack Propagation)
- Design approaches
- Recent numerical approaches and commercially important areas of composite materials, layered structures
- Extended Finite Element Method for Fracture Analysis of Structures

Learning outcomes of the course :

Through a deep understanding of the theory and the realization of a project, the student will be able to apply numerical tools to design structures and study crack propagation problems. In particular:

- He will have a deep understanding of fracture mechanics theories and will be able to summary, compare and explain them.
- He will have a deep understanding of the resolution methods of fracture mechanics problems, and will be able to summary, compare and explain them. He will also know their application range.
- He will understand mathematical treatment of crack tip fields.
- He will be able to apply the resolution methods to classical problems of fracture mechanics.
- He will be able to analyse and to evaluate (justify and criticise) these methods.
- He will be able to analyse new problems.

Prerequisites and co-requisites/ Recommended optional programme components :

Basic knowledge in

- Solid Mechanics
- Finite-element method
- Vector calculus
- Plasticity Theory

- Elasticity Theory
- Creep Theory

Planned learning activities and teaching methods :

Exercises with professor assistance and personal project.

Mode of delivery (face-to-face ; distance-learning) :

Face-to-Face

Required readings :

- Barenblatt G.I., Chorin A.J. Flow, Deformation and Fracture. Cambridge University Press, 2014.
- Sun C.T., Jin Z. Fracture Mechanics. Springer, 2012.
- Christensen R.M. The Theory of Materials Failure. Oxford: Oxford University Press, 2013. 296 p.
- Mathematical methods of fracture mechanics, L.V. Stepanova, Fizmatlit, Moscow, 2009.
- Mathematical methods of fracture mechanics, L.V. Stepanova, Samara State University, Samara, 2006.
- Fracture Mechanics. Lecture Course, Pestrikov V.M., Morozov E.M. Profession, St. Petersburg, 2012. 552 p.
- Lecture Notes on Fracture Mechanics, Alan T. Zehnder, Cornell University, Ithaca, 2010. 217 p. (http://ecommons.library.cornell.edu/handle/1813/3075)
- Fracture Mechanics: Fundamentals and applications, D. T. Anderson. CRC press, 1991.
- Hertzberg R.W., Vinci R.P., Hertzberg J.L. Deformation and Fracture Mechanics of Engineering Materials. Wiley, 2012. 784 p. ISBN 978-0470527801.
- Fang D., Liu J. Fracture Mechanics of Piezoelectric and Ferroelectric Solids. Springer, 2014. 420 p. ISBN 978-3642300868.
- Xiong .J., Sheonoi R.A. Fatigue and Fracture. Reliability and Fracture. London: Springer-Verlag, 2011. 222 p.

Assessment methods and criteria :

Evaluation is based on the realization of a project related to the use / development of numerical methods specific to fracture mechanics and on an examination.
The examination is based on the whole content of the class. Problems similar to the ones studied during the classes, and new problems will be part of the questions. Justification using the theoretical content is also asked.
Participation to the examination and achievement of the project are mandatory.

Teaching Method: Class participation is mandatory. Everyone is expected to participate in discussions relating to reading materials, homework, exams and lectures.

Guaranteed Recipe for Success:
1) Take notes during lecture and sections.
2) After each lecture but before the next lecture review your notes. Identify the parts you do not understand.
3) Come to each lecture and discussion section with specific questions.

4) Keep up with the reading so that you have some familiarity with each topic prior to hearing about it in the lecture.
5) Find at least one "partner" in the class with whom you can meet at least once or twice a week to discuss materials from the lectures, the reading assignments and the homework.
6) Take the homework assignment seriously. Do not try to do the whole assignment the night before it is due. Some version of the homework questions will appear on the exams.

Course Contents

Mechanics of brittle fracture

1 Introduction

1.1 Notable Fractures 1.2 Basic Fracture Mechanics Concepts 1.2.1 Small Scale Yielding Model 1.2.2 Fracture Criteria 1.3 Fracture Unit Conversions

2 Linear Elastic Stress Analysis of 2D Cracks

2.1 Notation 2.2 Introduction 2.3 Modes of Fracture

2.4 Mode III Field 2.4.1 Asymptotic Mode III Field 2.4.2 Full Field or Finite Crack in an Infinite Body 2.5 Mode I and Mode II Fields 2.5.1 Review of Plane Stress and Plane Strain Field Equations 2.5.2 Asymptotic Mode I Field 2.5.3 Asymptotic Mode II Field 2.6 Complex Variables Method for Mode I and Mode II Cracks

2.6.1 Westergaard Approach for Mode-I 2.6.2 Westergaard Approach for Mode-II 2.6.3 General Solution for Internal Crack with Applied Tractions 2.6.4 Full Stress Field for Mode-I Crack in an Infinite Plate 2.6.5 Stress Intensity Factor Under Remote Shear Loading 2.6.6 Stress Intensity Factors for Cracks Loaded with Tractions 2.6.7 Asymptotic Mode I Field Derived from Full Field Solution 2.6.8 Asymptotic Mode II Field Derived from Full Field Solution 2.6.9 Stress Intensity Factors for Semi-infinite Crack . 2.7 Some Comments 2.7.1 Three-Dimensional Cracks

3 Energy Flows in Elastic Fracture

3.1 Generalized Force and Displacement. 3.1.1 Prescribed Loads. 3.1.2 Prescribed Displacements. 3.2 Elastic Strain Energy. 3.3 Energy Release Rate, *G*. 3.3.1 Prescribed Displacement. 3.3.2 Prescribed Loads. 3.3.3 General Loading. 3.4 Interpretation of *G* from Load-Displacement Records. 3.4.1 Multiple Specimen Method for Nonlinear Materials. 3.4.2 Compliance Method for Linearly Elastic Materials. 3.4.3 Applications of the Compliance Method. 3.5 Crack Closure Integral for *G*. 3.6 *G* in Terms of *KI,KII,KIII* for 2D Cracks That Grow Straight Ahead. 3.6.1 Mode-III Loading. 3.6.2 Mode I Loading. 3.6.3 Mode II Loading. 3.6.4 General Loading (2D Crack). 3.7 Contour Integral for *G* (*J*-Integral). 3.7.1 Two Dimensional Problems. 3.7.2 Three-Dimensional Problems. 3.7.3 Example Application of *J*-Integral.

4 Criteria for Elastic Fracture

4.1 Introduction

4.2 Initiation Under Mode-I Loading. 4.3 Crack Growth Stability and Resistance Curve. 4.3.1 Loading by Compliant System. 4.3.2 Resistance Curve. 4.4 Mixed-Mode Fracture Initiation and Growth. 4.4.1 Maximum Hoop Stress Theory. 4.4.2 Maximum Energy Release Rate Criterion. 4.4.3 Crack Path Stability Under Pure Mode-I Loading. 4.4.4 Second Order Theory for Crack Kinking and Turning. 4.5 Criteria for Fracture in Anisotropic Materials. 4.6 Crack Growth Under Fatigue Loading. 4.7 Stress Corrosion Cracking.

5 Determining *K* and *G*

5.1 Analytical Methods. 5.1.1 Elasticity Theory. 5.1.2 Energy and Compliance Methods. 5.2 Stress Intensity Handbooks and Software. 5.3 Boundary Collocation. 5.4 Computational Methods: A Primer. 5.4.1 Stress and Displacement Correlation. 5.4.2 Global Energy and Compliance. 5.4.3 Crack Closure Integrals. 5.4.4 Domain Integral. 5.4.5 Crack Tip Singular Elements. 5.4.6 Example Calculations. 5.5 Experimental Methods. 5.5.1 Strain Gauge Method. 5.5.2 Photoelasticity. 5.5.3 Digital Image Correlation. 5.5.4 Thermoelastic Method.

6 Fracture Toughness Tests

6.1 Experimental Methods of Fracture Mechanics. 6.2 ASTM Standard Fracture Test. 6.2.1 Test Samples. 6.2.2 Equipment. 6.2.3 Test Procedure and Data Reduction. 6.3 Interlaminar Fracture Toughness Tests . 6.3.1 The Double Cantilever Beam Test. 6.3.2 The End Notch Flexure Test. 6.3.3 Single Leg Bending Test. 6.4 Indentation Method. 6.5 Chevron-Notch Method. 6.5.1 *KIV*

*Me*asurement. 6.5.2 *KIV* Measurement. 6.5.3 Work of Fracture Approach. 6.6 Wedge Splitting Method. 6.7 *K–R* Curve Determination. 6.7.1 Specimens. 6.7.2 Equipment. 6.7.3 Test Procedure and Data Reduction. 6.7.4 Sample *K–R* curve.

7 Elastic Plastic Fracture: Crack Tip Fields

7.1 Plasticity Zone near the Crack Tip. 7.2 Strip Yield (Dugdale) Model. 7.2.1 Effective Crack Length Model. 7.3 A Model for Small Scale Yielding. 7.4 Introduction to Plasticity Theory 7.5 Anti-plane Shear Cracks in Elastic-Plastic Materials in Small Scale Yielding (SSY). 7.5.1 Stationary Crack in Elastic-Perfectly Plastic Material. 7.5.2 Stationary Crack in Power-Law Hardening Material. The Hutchinson-Rice-Rosengren solution. 7.5.3 Steady State Growth in Elastic-Perfectly Plastic Material. 7.5.4 Transient Crack Growth in Elastic-Perfectly Plastic Material. 7.6 Mode-I Crack in Elastic-Plastic Materials. 7.6.1 Stationary Crack in a Power Law Hardening Material. 7.6.2 Slip Line Solutions for Rigid Plastic Material. 7.6.3 Large Scale Yielding (LSY) Example. 7.6.4 SSY Plastic Zone Size and Shape 7.6.5 CTOD-J Relationship. 7.6.6 Growing Mode-I Crack. 7.6.7 Three Dimensional Aspects. 7.6.8 Effect of Finite Crack Tip Deformation on Stress Field.

8 Elastic Plastic Fracture: Energy and Applications

8.1 Energy Flows. 8.1.1 When Does $G = J$? 8.1.2 General Treatment of Crack Tip Contour Integrals. 8.1.3 Crack Tip Energy Flux Integral. 8.2 Fracture Toughness Testing for Elastic-Plastic Materials. 8.2.1 Samples and Equipment. 8.2.2 Procedure and Data Reduction. 8.2.3 Examples of *J –R* Data. 8.3 Calculating *J* and Other Ductile Fracture Parameters. 8.3.1 Computational Methods. 8.3.2 *J* Result Used in ASTM Standard *JIC* Test. 8.3.3 Engineering Approach to Elastic-Plastic Fracture Analysis. 8.4 Fracture Criteria and Prediction. 8.4.1 *J* Controlled Crack Growth and Stability. 8.4.2 *J –Q* Theory. 8.4.3 Crack Tip Opening Displacement, Crack Tip Opening Angle
8.4.4 Cohesive Zone Model.

9. Three-dimensional crack problems

9.1 Fundamental tensors in elastostatics. The Kelvin-Somigliana tensor. The Kupradze-Bashelishvili tensor. 9.2. Fundamental theorems in elastostatics. Solution of the Neumann boundary value problem. Solution of the Dirichlet boundary value problem. Direct methods using Kelvin-Somigliana's tensor. 9.3. The symmetric opening mode I. The shear modes. 9.4. A planar crack in a bounded elastic medium. Singularity analysis. Solutions to some crack problems. The angular crack in an unbounded elastic medium. The edge crack in an elastic half-space. 9.5. On some mathematical methods for BIE in 3D. The Kupradze elastic potential theory. On the regularization of hypersingular integrals.

Recommended Literature

1. Stepanova L.V. *Mathematical Methods of Fracture Mechanics*. Moscow: Fizmatlit, 2009.
2. Zehnder A.T. Fracture Mechanics. Springer London Dordrecht Heidelberg New York : Springer, 2012. 223 p.
3. J.R. Rice, in *Fracture an Advanced Treatise*, vol. 2, ed. by H. Liebowitz (Academic Press, New York, 1968), pp. 191–311, chap. 3
4. Lawn B. *Fracture of Brittle Solids*, 2nd edn. Cambridge University Press: Cambridge, 1993.
5. Suresh S. *Fatigue of Materials*, 2nd edn. Cambridge University Press: Cambridge 1998.
6. M. Janssen, J. Zuidema, R.Wanhill, *Fracture Mechanics*, 2nd edn. London: Spon Press, 2004.
7. T.L. Anderson, *Fracture Mechanics Fundamentals and Applications*, 2nd edn. Boca Raton: CRC Press, 1995.
8. Sanford R.J. *Principles of Fracture Mechanics* New York : Prentice Hall, 2003.
9. Hellan K. *Introduction to Fracture Mechanics* New York: McGraw-Hill, 1984.
10. Broberg K.B. *Cracks and Fracture* San Diego: Academic Press, 1999.
11. Murakami Y. Metal *Fatigue: Effects of Small Defects and Nonmetallic Inclusions*. Oxford: Elsevier, 2002. 390 p.
12. Murakami S. *Continuum Damage Mechanics A Continuum Mechanics Approach to the Analysis of Damage and Fracture*. Springer, Dordrecht, Heidelberg, London, New York: Springer, 2012. 423 p.
13. Krupp U. *Fatigue Crack propagation in metals and alloys. Microstructural Aspects and Modelling Concepts*. Weinheim: Willey, 2007. 314 p.

14. Pestrikov V.M., Morozov V.M. Fracture Mechanics.Lecture Course. St. Petersburg: Profession, 2012. 552 p.
15. Liu A.F. Mechanics and Mechanisms of Fracture: An Introduction. Ohio: ASM International Materials Park, 2005. 467 p.
16. Mirzaei M. Fracture Mechanics. Lecture Notes. mmirzaei@modares.ac.ir http://www.modares.ac.ir/eng/mmirzaei
17. Aliabadi, M.H., Rooke, D.P.: Numerical Fracture Mechanics. Computation Mechanics, 296 pp. (2008).
18. Bui H.D. Mecanique de la rupture fragile. Paris: Masson Paris, 1978.
19. Bui H.D. Fracture Mechanics: Inverse problems and Solutions. Dordrecht: Springer, 2006. 376 p.
20. Mohammadi S. Extended Finite Element Method for Fracture Analysis of Structures. Willey, 2008. 280 p.
21. Bouharova T., Elboujdaini M., Pluvinage G. Damage and Fracture Mechanics: Failure Analysis of Engineering materials and Structures. Springer, 2009. 614 p.

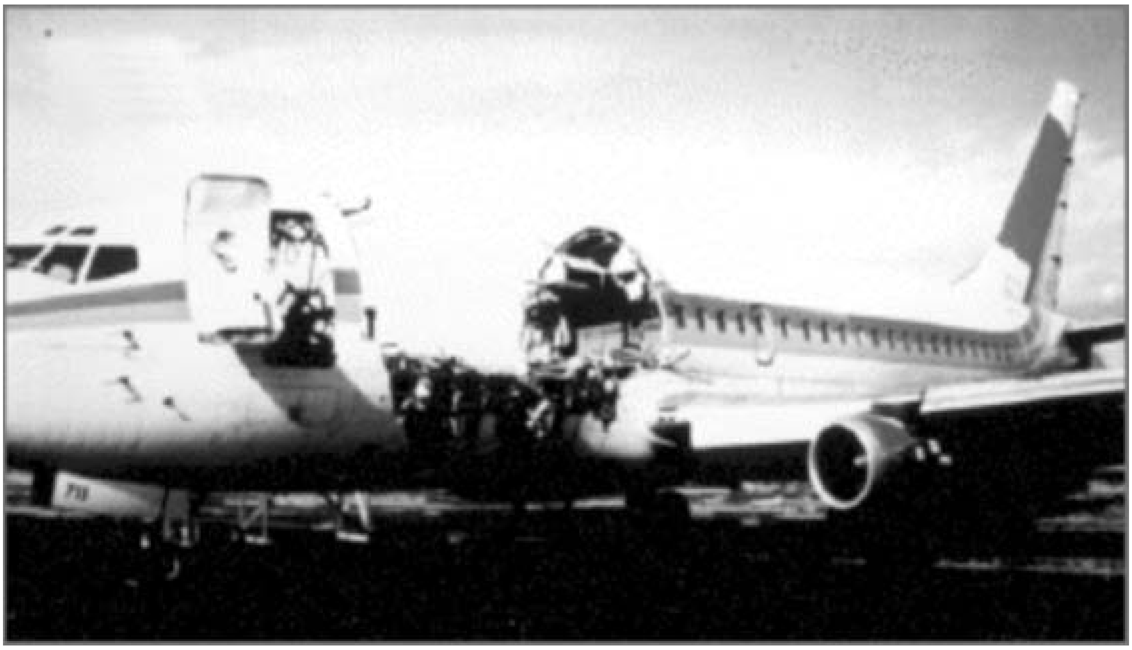
Examples of Exercises

1. Consider an aluminum plate loaded in tension. Suppose that the fracture toughness of this alloy is $KIC \approx 60$ MPa√m and the yield stress is $\sigma y = 400$ MPa.

(a) If a tensile stress of $\sigma a = 200$ MPa is applied what is the critical crack length, i.e. at what value of a is $KI = KIC$? At this critical crack length, estimate the size of the crack tip plastic zone using the relation $rp = 1\ \pi\ K2\ I\ \sigma 2y$. Are the SSY conditions satisfied in this case?

2. Glass is a strong but very brittle material. Typically $KIC \approx 1$ MPa√m for glass. If the plate described above was made of glass and loaded in tension with $\sigma a = 200$ MPa, what would the critical crack length be?

Consider the important cases of fracture shown below.



In-flight separation of an upper section of the fuselage of a B737-200 aircraft in 1988 attributed to corrosion and fatigue

The recommended literature and the brief description of the books.

1. Fang D., Liu J. Fracture Mechanics of Piezoelectric and Ferroelectric Solids. Springer, 2014. 420 p. ISBN 978-3642300868.

Fracture Mechanics of Piezoelectric and Ferroelectric Solids presents a systematic and comprehensive coverage of the fracture mechanics of piezoelectric/ferroelectric materials, which includes the theoretical analysis, numerical computations and experimental observations. The main emphasis is placed on the mechanics description of various crack problems such static, dynamic and interface fractures as well as the physical explanations for the mechanism of electrically induced fracture. The book is intended for postgraduate students, researchers, and engineers in the fields of solid mechanics, applied physics, material science and mechanical engineering.

2) Sun C.T., Jin L. Fracture Mechanics. Springer, 2012.

Most design engineers are tasked to design against failure, and one of the biggest causes of product failure is failure of the material due to fatigue/fracture. From leading experts in fracture mechanics, this new book provides new approaches and new applications to advance the understanding of crack initiation and propagation. With applications in composite materials, layered structures, and microelectronic packaging, among others, this timely coverage is an important resource for anyone studying or applying concepts of fracture mechanics.

- Concise and easily understood mathematical treatment of crack tip fields (chapter 3) provides the basis for applying fracture mechanics in solving practical problems
- Unique coverage of bi-material interfacial cracks (chapter 8), with applications to commercially important areas of composite materials, layered structures, and microelectronic packaging
- A full chapter (chapter 9) on the cohesive zone model approach, which has been extensively used in recent years to simulate crack propagation
- A unified discussion of fracture criteria involving nonlinear/plastic deformations

3) Hertzberg R.W., Vinci R.P., Hertzberg J.L. Deformation and Fracture Mechanics of Engineering Materials. Wiley, 2012. 784 p. ISBN 978-0470527801.

Deformation and Fracture Mechanics of Engineering Materials provides a combined fracture mechanics-materials approach to the fracture of engineering solids with comprehensive treatment and detailed explanations and references, making it the perfect resource for senior and graduate engineering students, and practicing engineers alike.

The book includes new end-of-chapter homework problems, examples, illustrations, and a new chapter on products liability and recall addressing the associated social consequences of product failure. The new edition continues to discuss actual failure case histories, and includes new discussion of the fracture behavior and fractography of ceramics, glasses, and composite materials, and a section on natural materials including bone and sea shells.