CREEP THEORY

Course contents:

The class presents the fundamentals of creep theory. The course is intended to summarize our present knowledge of creep phenomena and the existing creep theories. The course presents a comprehensive and critical overview of the creep theories. Fresh perspectives are given, highlighting central analytical methods and their applications as well as general theoretical and experimental results and their practical applications. Fundamental concepts are introduced, considering the specific questions of creep theories as well as general mathematical models and analytical methods. Physical and engineering aspects are also discussed. A comprehensive and coherent interpretation of all the phenomena related to creep rupture is presented. A thorough analysis of the relevant observations and a closed-packed presentation of the associated physical concepts introduce the listener to each one of the problem areas addressed in the course. Theories and experiments relating to the creep crack growth and creep-fatigue crack growth are either described.

Subjects covered by the lectures are

- Foundations of Creep Theory
- The Creep Curve
- Primary Creep
- Secondary Creep (Creep Potential Hypothesis, Isochoric Creep Behavior, Creep Parameters, Second-Order Effects)
- Tertiary Creep (Uniaxial Tertiary Creep, Multiaxial Tertiary Creep)
- Creep Behavior of Thick-Walled Tubes
- The Creep Potential Hypothesis in Comparison with the Tensor Function Theory
- Damage Mechanics
- Tensorial Generalization of Uniaxial Creep Laws to Multiaxial States of Stress
- Viscous Fluids
- Memories Fluids
- Viscoelastic Materials
- Viscoplastic Materials
- Creep and Damage Experiments

Learning outcomes of the course :

Through a deep understanding of the theory and the realization of a project, the student will be able to apply theoretical, asymptotical and numerical tools to solve creep theory problems. In particular:

- He will have a deep understanding of creep theories and will be able to summary, compare and explain them.
- He will have a deep understanding of the resolution methods of creep theories problems, and will be able to summary, compare and explain them. He will also know their application range.
- He will be able to apply the resolution methods to classical problems of creep theories.
- He will be able to analyze and to evaluate (justify and criticise) these methods.
- He will be able to analyze new problems.

Prerequisites and co-requisites/ Recommended optional programme components :

Basic knowledge in

- Tensor analysis
- Vector calculus

- Plasticity Theory
- Elasticity Theory
- Fracture Mechanics
- Damage Mechanics

Planned learning activities and teaching methods :

Exercises with professor assistance and personal project.

Mode of delivery (face-to-face ; distance-learning) :

Face-to-Face

Required readings :

- Findley W.N., Lai J.S., Onaran K. Creep and Relaxations of Nonlinear Viscoelastic Materials. Dover Publications, 2011. 400 p.
- Christensen R.M. Theory of Viscoelasticity. Dover Publications, 2010. 384 p.
- Betten J. Creep Mechanics. Berlin, Heidelberg, New York: Springer, 2005. 353 p.
- Riedel H. Fracture at High Temperature. Berlin, Heidelberg, New York: Springer-Verlag, 1986. 418 c.
- Rabotnov Y.N. Creep Problems in Structural Members. Amsterdam/London: North-Holland. 1969.
- Stepanova L.V. Mathematical methods of fracture mechanics. Samara: Samara State University, 2006.
- Riedel H. Crack Initiation and Growth. Encyclopedia of Materials: Science and Technology, 2008, Pages 1767-1773.
- Lemaitre J, Desmorat R. Engineering damage mechanics. Berlin: Springer, 2005.
- Murakami S. Continuum Damage Mechanics. A Continuum Mechanics Approach to the Analysis of Damage and Fracture. Dordrecht Heidelberg London New York: Springer, 2012. 423 p.
- Kubo S., Kitamura T. Creep-fatigue crack growth. Comprehensive Structural Integrity, 2007, Chapter 5.06, *Pages 273-307*
- Marques S.P.C., Creus G.J. Computational Viscoelasticity. New York: Springer, 2012. 124 p.

Assessment methods and criteria :

Evaluation is based on the realization of a project related to the use / development of numerical methods specific to creep mechanics and on an examination.
The examination is based on the whole content of the class. Problems similar to the ones studied during the classes, and new problems will be part of the questions. Justification using the theoretical content is also asked.
Participation to the examination and achievement of the project are mandatory.

Teaching Method: Class participation is mandatory. Everyone is expected to participate in discussions relating to reading materials, homework, exams and lectures.

Guaranteed Recipe for Success:
1) Take notes during lecture and sections.
2) After each lecture but before the next lecture review your notes. Identify the parts you do not understand.

3) Come to each lecture and discussion section with specific questions.
4) Keep up with the reading so that you have some familiarity with each topic prior to hearing about it in the lecture.
5) Find at least one "partner" in the class with whom you can meet at least once or twice a week to discuss materials from the lectures, the reading assignments and the homework.
6) Take the homework assignment seriously. Do not try to do the whole assignment the night before it is due. Some version of the homework questions will appear on the exams.

Course Contents

Creep Mechanics

1. Creep Behaviour of Isotropic and Anisotropic Materials; Constitutive Equations
1.1. Primary Creep
1.2.Secondary Creep
    1.2.1 Creep Potential Hypothesis
    1.2.2 Isochronic Creep Behavior
    1.2.3 Creep parameters
    1.2.4 Second-Order Effects
1.3. Tertiary Creep
    1.3.1 Uniaxial Tertially Creep
    1.3.2 Multiaxial Tertially Creep
2. Creep Behavior of Thick-Walled Tubes
2.1. Methods to describe the Kinematics
2.2. Isochoric Creep Behavior
2.3. Stress Field
2.4 Expansion and Failure Times
2.5 Numerical Computation and Examples
3. The Creep Potential Hypothesis in Comparison with the Tensor Function Theory
3.1 Isotropy
3.2. Oriented Solids
3.3 Modification of the Normality Rule
3.4. Anisotropy expressed through a Fourth-Rank Tensor
3.4.1. Irreducible Sets of Tensor Generators and Invariants
3.4.2. Special Formulations of Constitutive Equations and Creep Criteria
3.4.3. Incompressibility and Volume Change
3.4.4.Characteristic Polynomial for a Fourth Order Tensor
3.4.5. Lagrange Multiplier Method
3.4.6. Combinatorial Method
3.4.7. Simplified Representations
4. Damage Mechanics
4.1. Damage Tensor and Tensor of Continuity
4.2. Stresses in a Damage Continuum
4.3. Damage Effective Stress Concepts
5. Tensorial Generalization of Uniaxial Creep Laws to Multiaxial States of Stress
5.1.Polynomial Representation of Tensor Functions
5.2. Interpolation Methods for Tensor Functions
5.3. Tensorial Generalization of a Norton-Balley's Creep Law
5.4. Tensorial Generalization of a Creep Law including Damage
6. Viscous Fluids
6.1. Linear Viscous Fluids
6.2. Nonlinear Viscous Fluids
7. Memory Fluids
7.1.Maxwell Fluid

7.2. General Principle
7.3. Effects of Normal Stresses
8. Viscoelastic Materails
8.1. Linear Theory of Viscoelasticity
8.2. Nonlinear Theory of Viscoelsticity
8.3. Special Viscoelastic Models
8.3.1. Creep Spectra and Creep Functions for the Generalized Kelvin Model

8.3.2. Creep Behavior due to the $\sqrt{t}$ -Law
8.3.3. Creep as a Diffusion Process
8.3.4. Relaxation Spectra and Relaxation Functions for the Generalzed Maxwell Model

8.3.5. Relaxation Behavior due to the $\sqrt{t}$ -Law
8.3.6. Mechanical Hysteresis of Rheological Models
8.3.7. Complex Parameters of Rheological Models
8.3.8. Burgers Model
9. Viscoplastic Materials
9.1. Linear Theory of Viscoplasticity
9.2. Nonlinear Theory of Viscoplasticity
9.3. Viscoplasctic Behavior of Metals
10. Creep and Damage Experiments

Further Readings
1. Betten J. Creep Mechanics. Berlin, Heidelberg, New York: Springer, 2005. 353 p.
2. Riedel H. Fracture at High Temperature. Berlin, Heidelberg, New York: Springer-Verlag, 1986. 418 c.
3. Rabotnov Y.N. Creep Problems in Structural Members. Amsterdam/London: North-Holland. 1969.
4. Stepanova L.V. Mathematical methods of fracture mechanics. Samara: Samara State University, 2006.
5. Riedel H. Crack Initiation and Growth. Encyclopedia of Materials: Science and Technology, 2008, Pages 1767-1773.
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7. Murakami S. Continuum Damage Mechanics. A Continuum Mechanics Approach to the Analysis of Damage and Fracture. Dordrecht Heidelberg London New York: Springer, 2012. 423 p.
8. Kubo S., Kitamura T. Creep-fatigue crack growth. Comprehensive Structural Integrity, 2007, Chapter 5.06, Pages 273-307
9. Meyers M.A., Chawle K.K. Mechanical Behavior of Materials. New York: Cambridge University Press, 2009. 880 p.
10. Popov E. P. *Engineering Mechanics of Solids.* Englewood Cliffs, NJ: Prentice Hall, 1990.
11. H.F. Brinson, L.C. Brinson, Polymer Engineering Science and Viscoelasticity: An introduction. New York: Springer, 2008.
12. E.J. Barbero, Finite Element Analysis of Composite Materials. Boca Raton: CRC Press, 2008.
13. K.F. Riley, M.P. Hobson, S.J. Bence, Mathematical Methods for Physics and Engineering. New York: Cambridge University Press, 2006.
14. G.A. Holtzapfel, Nonlinear Solid Mechanics. West Sussex: Wiley, 2004.
15. C. Truesdell, W. Noll, The Non-Linear Field Theories of Mechanics. Berlin: Springer, 2004.
16. O.C. Zienkiewicz, R.L. Taylor, RL Finite Element Method for Solid and Structural Mechanics. Oxford: Elsevier Butterworth-Heinemann, Jordan Hill, 2005.
17. Ibrahimbegovic A. Nonlinear Solid Mechanics. Theoretical Formulations and Finite Element Solution Methods. New York, Springer, 2009. 588 p.

Problems and Solutions (Meyers M.A., Chawle K.K. Mechanical Behavior of Materials. New York: Cambridge University Press, 2009. 880 p.)

Exercises

1.1 A jet turbine rotates at a velocity of 7,500 rpm. Calculate the stress acting on the turbine blades if the turbine disc radius is 70 cm and the cross-sectional area is 15 cm2. Take the length to be 10 cm and the alloy density to be 8.5 g/cm3.

1.2 The material of the jet turbine blade in Problem 1.1, Superalloy IN 718, has a room-temperature yield strength equal to 1.2 GPa; it decreases with temperature as

$$\sigma = \sigma_0 \left(1 - \frac{T - T_0}{T_m - T_0} \right)$$

where T_0 is the room temperature and T_m is the melting temperature in K ($T_m = 1,700$ K). At what temperature will the turbine flow plastically under the influence of centripetal forces?

1.3

(a) Describe the mechanical properties that are desired in a tennis racket, and recommend different materials for the different parts of the racket.

(b) Describe the mechanical properties that are desired in a golf club, and recommend different materials for the different parts of the club.

1.4 On eight cubes that have one common vertex, corresponding to the origin of axes, draw the family of {111} planes. Show that they form an octahedron and indicate all <110> directions.

1.5 The frequency of loading is an important parameter in fatigue. Estimate the frequency of loading (in cycles per second, or Hz) of an automobile tire in the radial direction when the car speed is 100 km/h and the wheel diameter is 0.5 m.

1.6 Indicate, by their indices and in a drawing, six directions of the <112> family.

1.7 The density of Cu is 8.9 g/cm3 and its atomic weight (or mass) is 63.546. It has the FCC structure. Determine the lattice parameter and the radius of atoms.

1.8 The lattice parameter for W(BCC) is $a = 0.32$ nm. Calculate the density, knowing that the atomic weight (or mass) of W is 183.85.

1.9 Consider the unit cell of the CsCl which has NaCl structure. The radius of Cs+ is 0.169 nm and that of Cl is 0.181 nm. (a) Determine the packing factor of the structure, assuming that Cs+ and Cl− ions touch each other along the diagonals of the cube. (b) Determine the density of CsCl if the atomic weight of Cs is 132.905 and of Cl is 35.453.

1.10 MgO has the same structure as NaCl. If the radii of O2− and Mg2+ ions are 0.14 nm and 0.070 nm, respectively, determine (a) the packing factor and

(b) the density of the material. The atomic weight of O2 is 16 and that of Mg is 24.3.

1.11 Germanium has the diamond cubic structure with interatomic spacing of 0.245 nm. Calculate the packing factor and density. (The atomic weight of germanium is 72.6.)

1.12 The basic unit (or mer) of polytetrafluoroethylene (PTFE) or Teflon is C2F4. If the mass of the PTFE molecule is 45,000 amu, what is the degree of polymerization?

1.13 Using the representation of the orthorhombic unit cell of polyethylene, calculate the theoretical density. How does this value compare with the density values of polyethylene obtained in practice?

1.14 A pitch blend sample has five different molecular species with molecular masses of 0.5 Ч 106, 0.5 Ч 107, 1 Ч 107, 4 Ч 107, and 6 Ч 107. Compute the number-averaged molecular weight and weight-averaged molecular weight of the sample.

1.15 Different polymorphs of a material can have different mechanical properties. Give some examples.

Exercises

2.1 A cylindrical specimen creeps at a constant rate during 10,000 hours when it is subjected to a constant load of 1,000 N. The initial diameter and length of the specimen are 10 and 200 mm, respectively, and the creep rate

is 10−8 h−1. Find:
(a) The length of the specimen after 104, 106, and 108 hours.
(b) The true and engineering strains after these periods.
(c) The true and engineering stresses after these periods.

2.2 Give three reasons why the extrapolation of creep data obtained over a short period can be dangerous over long periods.

2.3 By means of plots, show how isochronal stress-versus-strain curves can be constructed from creep curves for various stresses at a certain temperature.

2.4. Assuming that pure silver creeps according to the Dorn equation, estimate the rupture time at 400 °C when the silver is subjected to a stress of 50 MPa, knowing that at 300 °C and at the same stress level the rupture time is 2,000 hours.

2.5. An amorphous polymer has a glass transition temperature of 100 °C. A creep modulus of 1 GPa was measured after 1 hour at 75 °C. Using the Williams--Landel--Ferry expression, determine the time required to reach this modulus at 50 °C.

2.6 The viscosity of an amorphous polymer is 105 Pa ·s at 190 °C and 2 Ч 102 Pa · s at 270 °C. At what temperature will the viscosity be 109 Pa · s?

2.7 What is the strain undergone by a polymer in tension at 67 °C for one minute if the polymer's strain-rate response is given by $\dot{\varepsilon}$ = 4.5 Ч 1028 exp (−200 kJ/RT)?

2.8 A nylon cord has an initial stress of 2 MPa and is used to tie a sack. If the relaxation time for this cord is 250 days, how many days will it take for the stress to drop to 0.1 MPa?

2.9 How much time will it take for a rod of polymer to extend from 20 mm to 30 mm at 120 °C if it is deformed at a strain rate $\dot{\varepsilon}$ = 4.5 Ч 1011 exp(-100 kJ/RT)?

2.10 Find the initial stress for a nylon cord if the relaxation time for the cord is 100 days and in 50 days the stress is reduced to 1 MPa.

2.11 (a) Determine the strain-rate sensitivity in the superplastic range for the alloys shown in Figures 13.39(a) and 13.40, and explain the values encountered. (b) Why does the maximum in ductility vary with temperature in Figure 13.40?

2.12 Explain why the presence of voids decreases the maximum strain in superplastic deformation.

2.13 A polymer has a viscosity of 1012 Pa ·s at 150 °C. If this polymer is subjected to a tensile stress of 100 MPa at that temperature, compute the deformation after 10 h. Assume the polymer to behave as a Maxwell solid. Take E = 5 GPa, and use the equation E = σ E + 1 3η σt.

2.14 For a given polymer, the activation energy for stress relaxation was measured to be 10 kJ/mol. If the stress relaxation time for this polymer at room temperature is 3,600 s, what would be the relaxation time at 100 °C?

2.15 In a laboratory experiment for potential creep deformation, it was found that the creep rate ($\dot{\varepsilon}$) of an alloy is 0.5% per hour at 780 °C and 2.8 Ч 10−2% per hour at 650 °C.
(a) What is the activatuion energy for creep in the given temperature range?
(b) At a temperature of 550 °C, what is the estimated creep rate?

2.16 A component used in a chemical plant is being used at 600 °C and stress of 25 MPa. The corresponding creep rate is: 3 Ч 10−12 s −1. If the stress is increased to 35 MPa and the temperature to 650 °C, what will be the corresponding creep rate?
Given:
Q = 150 kJ/mol,
n = 4.5 (stress exponent).

2.17 Honey has a viscosity of 1.5 Pa · s at room temperature. If the activation energy of honey is 20 kJ/mol, what will be its viscosity at 0 °C?

2.18 From the plot in Figure Ex13.29 on p. 711, determine the amount of strain that a titanium component will undergo if subjected to a stress of 30 MPa at a temperature of 700 °C for 1 hour.
Given:
$D0$ = 1 cm2 s−1,
G (shear modulus) = 43.8 GPa,
Q = 15 kJ/mol,
b = 0.3 nm.

2.19 Give an example of the creep phenomenon in nature (without interference of human paws).

2.20 Using the parameters in Figure 13.6, calculate the rupture time by the Larson--Miller equation: (a) at a temperature of 1089 K and stress of 100 MPa; (b) at a temperature of 1005 K and stress of 200 MPa; (c) at a temperature of 922 K and stress of 300 MPa.

2.21 At room temperature, the relaxation time for a polymer is 100 days. What will be its relaxation time if the activation energy is one-fourth of its current value?

2.22 Calculate the creep compliance for stainless steel with a relaxation time of 72 hours.

Example 1.11

Determine the maximum strain that the collagen fibers can experience without damage if their shape is as given in Figure 1.39 with a ratio between amplitude and wavelength of 0.2

We can assume a sine function of the form:

$$y = k \sin 2\pi x / \lambda.$$

The maximum of y is reached when $x = \pi/4$.
Hence:

$$y_{max} = k = \lambda/5.$$

We can integrate over the length of the sine wave from 0 to 2π. However, this will lead to an elliptical integral of difficult solution. A simple approximation is to consider the shape of the wavy protein as an ellipse with major axis $2a$ and minor axis $2b$. The circumference is given by the approximate expression (students should consult a mathematics text to obtain this expression):

$$L \approx \pi \left[\frac{3}{2}(a+b) - (ab)^{1/2} \right].$$

In the sine function, we have two arms, one positive and one negative. Their sum corresponds, in an approximate manner, to the circumference of the ellipse. The strain is equal to:

$$\varepsilon = \frac{L - 4a}{4a} = \frac{\pi \left[\frac{3}{2}(a+b) - (ab)^{1/2} \right] - 4a}{4a}.$$

Thus:

$$\varepsilon = \frac{\pi}{4} \left[\frac{3}{2}\left(1 + \frac{b}{a}\right) - \left(\frac{b}{a}\right)^{1/2} \right] - 1.$$

The following ratio is given:

$$\frac{b}{2a} = 0.2 \quad \text{and} \quad \frac{b}{a} = 0.4.$$

The corresponding strain is:

$$\varepsilon = 0.53.$$

Beyond this strain, the collagen will break.

Example 2.1

Calculate the material properties E, G, and ν of SiC, given the graphs of the longitudinal and shear sound velocities obtained using ultrasonic equipment. (See Figure E2.1). Here, $\rho = 3.18 \times 10^3$ kg/m³ and the length of specimen is $L = 4$ mm.

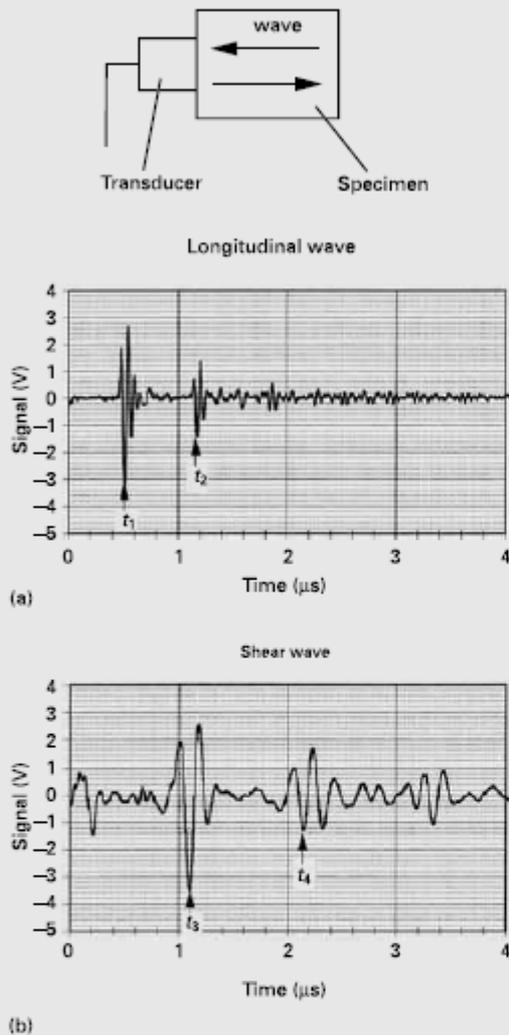Transducer   Specimen

Longitudinal wave

(a)

Shear wave

(b)

Fig. E2.1

Solution: We take equivalent peaks, marked by arrows, in sequential signal packets. We must remember that the pulse reflects at the free surface, and therefore, we have to take twice the length of the pulse. We have

$$V_\ell = \frac{2L}{t_2 - t_1} = \frac{2 \times 4 \times 10^{-3}}{(1.16 - 0.52) \times 10^{-6}} = 12.5 \times 10^3 \text{ m/s},$$

$$V_\ell = \sqrt{\frac{E}{\rho}},$$

$$E = \rho V_\ell^2 = 3.18 \times 10^3 \times (12.5 \times 10^3)^2 = 496.9 \times 10^9 \text{ Pa} = 496.9 \text{ GPa},$$

$$V_s = \frac{2L}{t_4 - t_3} = \frac{2 \times 4 \times 10^{-3}}{(2.15 - 1.10) \times 10^{-6}} = 7.62 \times 10^3 \text{ m/s}$$

$$V_s = \sqrt{\frac{G}{\rho}},$$

$$G = \rho V_s^2 = 3.18 \times 10^3 \times (7.62 \times 10^3)^2 = 184.6 \times 10^9 \, \text{Pa} = 184.6 \, \text{GPa}.$$

Since, according to Table 2.2,

$$G = \frac{E}{2(1+v)},$$

where v is Poisson's ratio, as explained in Section 2.4, it follows that

$$v = \frac{E}{2G} - 1 = \frac{496.9}{2 \times 184.6} - 1 = 0.346.$$

(*Note:* The preceding calculations were conducted assuming uniaxial stress and without the dispersion correction; hence, the results are only approximate.)

A correct equation for the elastic modulus would be

$$V_\ell = \sqrt{\frac{\bar{E}}{\rho}}, \quad \bar{E} = \frac{(1-v)}{(1+v)(1-2v)} E.$$

This is because the length of the pulse is much shorter than the lateral dimension of the specimen, and therefore, the specimen is stressed in uniaxial strain.[2]

Example 2.2

A bar of a material with Young's modulus, E, length, L, and cross-sectional area, A, is subjected to an axial load, P. Derive an expression for strain energy stored in the bar.

Solution: In order to determine the total elastic strain energy stored in a body under a general stress, σ, we need to integrate the elastic strain energy density over the original volume of the material. Thus, the total elastic strain energy for a material of volume V, can be written as

$$U_{\text{total}} = \int_V \frac{\sigma^2}{2E} \, dV.$$

For the simple case of a tensile force P acting on an area, A, we can write this as

$$U_{\text{total}} = \int_0^L \frac{P^2}{2EA^2} \, dx = \frac{P^2 L}{2EA^2}.$$

Example 2.3

A cylindrical steel specimen (length $=200$ mm, diameter $=5$ mm), is subjected to a torque equal to 40 N $\cdot$ m.

(a) What is the deflection of the specimen end, if one end is fixed?
(b) Will the specimen undergo plastic deformation?

Given:

$E = 210$ GPa,

$v = 0.3$,

$\sigma_y = 300$ MPa (tensile yield stress).

Solution:

(a) $\tau_{max} = \dfrac{T \cdot c}{J}$. (1)

Given

$$T = 40 \text{ N·m}, \quad c = \frac{d}{2} = 2.5 \text{ mm}.$$

To calculate τ_{max}, we need to know J:

$$J = \pi \frac{c^4}{2}. \tag{2}$$

Substitute (2) into (1).

$$\tau_{max} = \frac{T \cdot c}{\pi \left(\frac{c^4}{2}\right)} = \frac{2T}{\pi c^3} = \frac{2 \cdot 40}{\pi \cdot (2.5)^3} \frac{\text{N·m}}{\text{mm}^3}$$

$$= 1630 \text{ MPa}$$

$$= 1.63 \text{ GPa}.$$

Shear stress and shear strain are related as

$$\tau = G\gamma.$$

G can be calculated from E and v.

$$G = \frac{E}{2(1+v)} = \frac{210}{2(1+0.3)} = 81 \text{ GPa},$$

$$\gamma = \frac{\tau_{max}}{G} = \frac{1.63}{81} = 0.02.$$

But,

$$\gamma = \frac{c\theta}{L},$$

where θ is the angle of rotation.

$$\text{Torsional deflection} = \text{angle of rotation}$$

$$= \frac{\gamma L}{c}$$

$$= \frac{0.02 \times 200}{2.5} = 1.6 \text{ radians}.$$

(b) $\tau_{max} = 1.63$ GPa.

The shear stress required to cause permanent deformation is related to the yield stress as follows:

$$\tau_y = \frac{\sigma_y}{2}.$$

When the stress is σ_y,

$$\tau_y = \frac{300}{2} = 150 \text{ MPa}.$$

But from (a), $\tau_{max} = 1.63$ GPa > 150 MPa. Therefore, the specimen will undergo plastic deformation.

Example 13.1

The alloy INCONEL 718 was diligently tested by graduate student M. A. Meyers at 820 MPa and a temperature of 650 °C. Three conditions of the alloy were tested: undeformed, cold-rolled, and shock-hardened (by explosives). After days of patient data collecting (this was in the 1970s prior to automated data recording), he obtained the curves shown in Figure E13.1. Using the Larsen–Miller parameter, determine the times to rupture if this alloy will be used at (a) 550 °C and the same stress and (b) 650 °C and 600 MPa. Take $C = 18$.
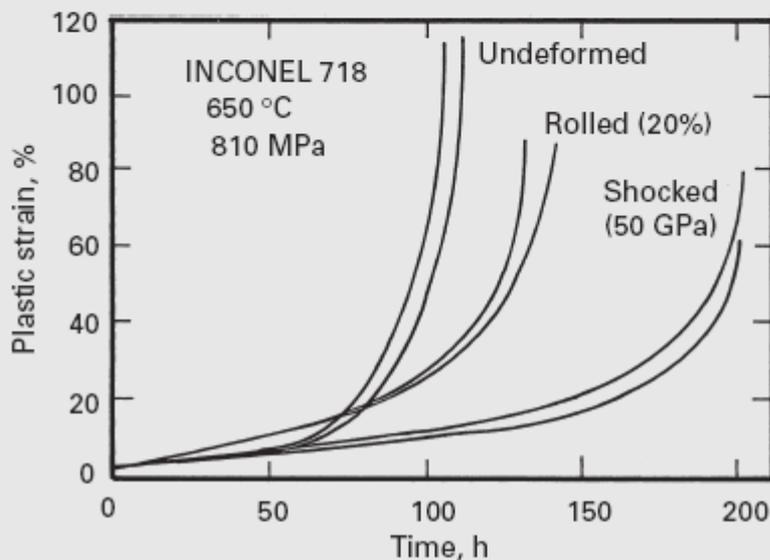

Fig. E13.1

Solution:

(a) We use the equation

$$T(\log t_r + C) = m$$

with $t_r \approx 110$ hours for the undeformed condition, $t_r \approx 130$ hours for the rolled condition, and $t_r \approx 200$ hours for the shocked condition. We have

$T = 650 + 273 = 923$ K

$923(\log 110 + 18) = 17{,}575,$

$923(\log 130 + 18) = 17{,}642,$

$923(\log 200 + 18) = 17{,}814.$

At 550 °C, $T = 823$ K, and $t_r = 22.6 \times 10^4$ hours for the undeformed condition, $t_r = 27.3 \times 10^3$ hours for the rolled condition, and $t_r = 44.2 \times 10^3$ hours for the shocked condition.

(b) No result can be obtained in this case because the stress has to be constant, for the application of the Larsen--Miller equation to two conditions.

Exercises.

Example 1 Viscoelastic rod subjected to constant axial load (Abaqus/Standard Example Problems Manual, Volume I).

This is a simple but interesting problem that introduces Abaqus viscoelastic procedures and explains how to find the parameters needed and to compare the answer with an analytical solution. We present it here with small modifications to adapt it to SI standards. The rod has a length of 300 mm, and a square section with 30 mm in each side. A constant axial stress of 10 MPa is applied suddenly at one end and maintained constant. We aim the history of deformations in time. It is here assumed that the reader has a general knowledge of Abaqus/CAE (a tutorial ''Getting started with Abaqus interactive edition'' is available with the software). We will show here only the details related to the viscoelastic analysis. Determination of material parameters: The material is represented by a Zener model with an extensional relaxation function.

Entering data into Abaqus: we assume that the student has a basic familiarity with Abaqus for the analysis of elastic problems. Thus, we include here only the details related to viscoelastic analyses. The example is called BeamV. The characteristics of the material are given in module Property.

1. Entering the module, we go to Create material. The Edit Material dialog box appears.

2. Name the material Polymer.

3. From the Material Behavior menu select Mechanical-Elasticity-Elastic. Abaqus displays the Elastic Data. Maintain Isotropic.

4. In Moduli Timescale (for viscoelasticity) choose instantaneous.

5. Type the value 1000 for Young modulus and the value 0.4833 for Poisson ratio in the respective fields.

6. Without leaving this box, go again to Mechanical, elasticity, viscoelastic. In Domain choose time, in time Prony. Inform gi:0.901, ki:0, tau:98.99. If we have more terms in the Prony series we enter them below.

7. Save.

Defining and assigning section properties: Still in Module Property, select the tool Create Section to define a Bar section:

1. The Create Section dialog box appears.

2. In the Create Section dialog box:

2.1. Name the section BarVSection.

2.2. Accept Category: Solid.

2.3. Type: Homogeneous.

2.4. Click Continue. The Edit Section dialog box appears.

3. In the Edit Section dialog box:

3.1. In Material accept Polymer. If you had defined other materials, you could click the arrow next to the Material text box to see a list of available materials and to select the material of your choice.

3.2. Click OK.

Go to Assign Section, to assign the section BarSection, to the model.

1. Click with the mouse on the beam and press DONE under the viewport.

2. In the Section Assignement box verify Section: BeamVSection.

3. Click OK. The bar changes color.

Defining the assembly: This module may be used to assemble a complex model from its parts. In the present case, we have only one part. In the Module list located below the toolbar, click Assembly to enter the Assembly module.

1. From the menu bar, select Instance Part. The Create Instance dialog box appears.

2. In the Instance Type dialog box, choose Independent (mesh on instance).

3. In the dialog box, select BeamV, click OK.

Configuring the Analysis: Now that the assembly has been created, you can move to the Step module to configure your analysis. In the Module list, click Step to enter the Step module. Select Create Step.

1. The Create Step dialog box appears with a list of all the general procedures and a step name. Write name stepvisco

2. Select Procedure type: General/Visco and click Continue.

3. The Edit Step Box opens. In the Basic tab, select time period: 10000,

NLGeom-Off.

4. In the Incrementation tab, Maximum number: 1000; increment size: initial: 10, minimum: 0.001, maximum: 30. In the option creep/swelling/viscoelastic strain error tolerance: 0.005.

5. Click OK to create the step and to exit the Edit Step dialog box.

Then, we have to set the boundary conditions encastré in one of the ends and apply a pressure -10 to the other, mesh the bar, choose the element type and solve.

Comparing the result to the analytical solution.

Example 2 Change the integration parameters in Example 1 to see their influence on the approximation.

Example 3 Apply a constant displacement of 5 mm to the free end of the bar and determine the relaxation curve.

Example 4 Thick cylindrical shell under internal pressure-Plane strain.