Damage Mechanics

Course contents:

The class presents the basics of damage mechanics. This course aims to give a concise overview of the current state of damage mechanics, and then to show the fascinating possibility of this promising branch of solid mechanics. Damage mechanics which originally started as a phenomenological theory of damage and fracture has been reinforced logically by the help of well established theoretical frameworks of material science, nonlinear continuum mechanics, irreversible thermodynamics, micromechanics, computational mechanics, etc., and now has been established as a precise and systematic discipline for damage and fracture analysis. The results of the development of this field of mechanics are presented in this course.

Subjects covered by the lectures are

- Foundations of Continuum Damage Mechanics
- Material Damage and Continuum Damage Mechanics
- Mechanical Representation of Damage and Damage Variables
- Thermodynamics of Damaged Material
- Inelastic Constitutive Equation and Damage Evolution
- Equation of Material with Anisotropic Damage
- Application of Continuum Damage Mechanics
- Fatigue Damage
- Creep Damage and Creep-Fatigue Damage
- Elastic-Brittle Damage
- Continuum Damage Mechanics of Composite Materials
- Local Approach to Damage and Fracture Analysis
- Damage variable in Fracture Mechanics

Learning outcomes of the course :

Through a deep understanding of the theory and the realization of a project, the student will be able to apply theoretical, asymptotical and numerical tools to solve damage mechanics problems. In particular:

- He will have a deep understanding of damage mechanics theories and will be able to summary, compare and explain them.
- He will have a deep understanding of the resolution methods of damage mechanics problems, and will be able to summary, compare and explain them. He will also know their application range.
- He will be able to apply the resolution methods to classical problems of damage mechanics.
- He will be able to analyse and to evaluate (justify and criticise) these methods.
- He will be able to analyse new problems.

Prerequisites and co-requisites/ Recommended optional programme components :

Basic knowledge in

- Solid mechanics
- Finite-element method
- Vector calculus
- Plasticity Theory
- Elasticity Theory

- Creep Theory
- Fracture Mechanics

Planned learning activities and teaching methods :

Exercises with professor assistance and personal project.

Mode of delivery (face-to-face ; distance-learning) :

Face-to-Face

Required readings :

- Voyiadis G.Z., Kattan P.I. Damage Mechanics with Finite Elements: Practical Applications with Computer Tools. Springer, 2012. 113 p. ISBN 978-3642626753.
- Bouharova T., Elboujdaini M., Pluvinage G. Damage and Fracture Mechanics: Failure Analysis of Engineering materials and Structures. Springer, 2009. 614 p.
- Stepanova L.V. Mathematical methods of fracture mechanics. Moscow: Fizmatlit, 2009.
- Stepanova L.V. Mathematical methods of fracture mechanics. Samara: Samara State University, 2006.
- Pestrikov V.M., Morozov E.M. Fracture Mechanics. Lecture Course. St. Petersburg: Profession, 2012. 552 p.
- Lemaitre J, Desmorat R. Engineering damage mechanics. Berlin: Springer, 2005.
- Murakami S. Continuum Damage Mechanics. A Continuum Mechanics Approach to the Analysis of Damage and Fracture. Dordrecht Heidelberg London New York: Springer, 2012. 423 p.

Assessment methods and criteria :

Evaluation is based on the realization of a project related to the use / development of numerical methods specific to damage mechanics and on an examination.
The examination is based on the whole content of the class. Problems similar to the ones studied during the classes, and new problems will be part of the questions. Justification using the theoretical content is also asked.
Participation to the examination and achievement of the project are mandatory.

Course Contents
Damage Mechanics
Foundations of Continuum Damage Mechanics
1 Material Damage and Continuum Damage Mechanics
1.1 Damage and Its Microscopic Mechanisms. 1.1.1 Scales of Damage Phenomena. 1.1.2 Physical Mechanisms of Damage. 1.1.3 Damage in Fracture Problems. 1.2 Representative Volume Element and Continuum Damage Mechanics. 1.2.1 Representative Volume Element. 1.2.2 Notion of Continuum Damage Mechanics
2 Mechanical Representation of Damage and Damage Variables
2.1 Mechanical Modeling of Damage. 2.1.1 Modeling by Effective Area Reduction. 2.1.2 Modeling by the Variation in Elastic Modulus. 2.1.3 Modeling by Void Volume Fraction. 2.2 Mechanical Representation of Three-Dimensional Damage State. 2.2.1 Scalar Damage Variable. 2.2.2 Plural Scalar Damage Variables. 2.2.3 Vector Damage Variable. 2.2.4 Damage State Defined by Effective Area Reduction, Damage Tensor of Second-Order. 2.2.5 Damage State Defined by Geometrical Configuration of Microvoids. 2.2.6 Damage Sate Defined by Directional Distribution of Microvoid

Density. 2.2.7 Damage Tensors of Fourth- and Eighth-Order. 2.3 Effective Stress and Hypothesis of Mechanical Equivalence. 2.3.1 Effective Stress Tensors. 2.3.2 Damage Effect Tensors, Representation of Effective Stress Tensors. 2.3.3 Hypothesis of Strain Equivalence. 2.3.4 Hypothesis of Energy Equivalence 1 – Complementary Strain Energy Equivalence. 2.3.5 Hypothesis of Energy Equivalence 2 – Strain Energy Equivalence. 2.3.6 Hypothesis of Total Energy Equivalence. 2.4 Elastic Constitutive Equation and Elastic Modulus Tensor of Damaged Material – Comparison Between Results by Different Effective Stresses and Equivalence Hypotheses. 2.4.1 Elastic Constitutive Equations of Damaged Material. 2.4.2 Matrix Representation of Damage Effect Tensors. 2.4.3 Matrix Representation of Elastic Constitutive Equation and Elastic Modulus Tensor. 2.4.4 Elastic Constitutive Equation and Elastic Compliance Tensor 1 – By Hypothesis of Strain Equivalence. 2.4.5 Elastic Constitutive Equation and Elastic Compliance Tensor 2 – By Hypothesis of Complementary Strain Energy Equivalence. 2.4.6 Elastic Constitutive Equation and Elastic Compliance Tensor 3 – By Complementary Strain Energy Function and Representation Theorem. 2.5 Refinement of Continuum Damage Mechanics and Its Results.

3 Thermodynamics of Damaged Material

3.1 Thermodynamics of Continuum. 3.1.1 State Variables and Principle of Local State. 3.1.2 First Law of Thermodynamics. 3.1.3 Second Law of Thermodynamics and Clausius-Duhem Inequality. 3.1.4 Gibbs Relation and Thermodynamic Potentials. 3.2 Thermodynamic Constitutive Theory of Inelasticity With Internal Variables. 3.2.1 Thermodynamic Potentials and Constitutive Equations. 3.2.2 Dissipation Potentials and Evolution Equations of Internal Variables. 3.2.3 Constitutive Equations Expressed in Stress Space. 3.3 Extension of Thermodynamic Constitutive Theory of Inelasticity. 3.3.1 Generalized Standard Material. 3.3.2 Thermodynamic Constitutive Theory Based on Multiple Dissipation Potentials – Quasi-Standard Thermodynamic Approach.

4 Inelastic Constitutive Equation and Damage Evolution
Equation of Material with Isotropic Damage

4.1 One-Dimensional Inelastic Constitutive Equation of Material with Isotropic Damage. 4.1.1 Elastic-Plastic Deformation of Damaged Material. 4.1.2 Viscoplastic Deformation of Damaged Material. 4.2 Three-Dimensional Inelastic Constitutive Equations of Material with Isotropic Damage. 4.2.1 Internal Variables and Thermodynamic Constitutive Theory. 4.2.2 Thermodynamic Potential and Dissipation Potential. 4.2.3 Elastic-Plastic Constitutive Equation of Damaged Material. 4.2.4 Viscoplastic Constitutive Equation of Damaged Material. 4.2.5 Evolution Equation of Elastic-Plastic Damage and Viscoplastic Damage. 4.2.6 Threshold Value *pD* of Plastic Strain for Damage Initiation – In the Case of Fatigue Damage. 4.3 Strain Energy Release Rate and Stress Criterion for Damage Development in Elastic-Plastic Damage. 4.3.1 Strain Energy Release Rate Due to Damage Development. 4.3.2 Energy Dissipation in Elastic-Plastic Damage. 4.3.3 Stress Triaxiality and Stress Criterion for Damage Development. 4.3.4 Effect of Stress Sign on Damage Development. 4.3.5 Stress Criterion for Ductile Damage. 4.4 Inelastic Damage Theory Based on Hypothesis of Total Energy Equivalence. 4.4.1 Thermodynamic Potential and State Equation. 4.4.2 Dissipation Potential and Evolution Equation of Plastic Damage. 4.4.3 Dissipation Potential and Evolution Equation of Viscoplastic Damage.

5 Inelastic Constitutive Equation and Damage Evolution
Equation of Material with Anisotropic Damage

5.1 Elastic-Plastic Anisotropic Damage Theory Based on Second-Order Symmetric Damage Tensor. 5.1.1 Internal Variables and Thermodynamic Constitutive Theory. 5.1.2 Helmholtz Free Energy and Elastic Constitutive Equation. 5.1.3 Dissipation Potential Functions of Plastic Deformation and Damage. 5.1.4 Plastic Constitutive Equation and Damage Evolution Equation. 5.2 Elastic-Plastic Anisotropic Damage Theory in Stress Space. 5.2.1 Thermodynamic Constitutive Theory in Stress Space. 5.2.2 Gibbs Potential and Elastic Constitutive Equation. 5.2.3 Dissipation Potential Functions of Plastic Deformation and Damage. 5.2.4 Plastic Constitutive Equation and Damage Evolution Equation. 5.3 Fourth-Order Symmetric Damage Tensor and Its Application to Elastic-Plastic-Brittle Damage. 5.3.1 Constitutive and Evolution Equations for Elastic-Brittle Damage. 5.3.2 Opening-Closing Effect of Cracks in Brittle Damage Field and Its Mechanical Representation – Positive Projection Tensors for Strain and Stress. 5.3.3 Effective Elastic Modulus Tensor in Opening and Closing States of Cracks in Brittle Material. 5.3.4 Damage Variable Defined

Equation of Damage. 9.5.2 Unilateral Effect of Elastic Deformation. 9.5.3 Damage Surface and Loading Criterion. 9.5.4 Application to Elastic-Brittle Damage of Salem Limestone.

10 Continuum Damage Mechanics of Composite Materials

10.1 Damage of Laminate Composites. 10.1.1 Damage Variables and Thermodynamic Potential. 10.1.2 Evolution Equation of DamageVariables. 10.1.3 Plastic Constitutive Equation of Damaged Material. 10.1.4 Application to Laminate Composites. 10.1.5 Brittle Damage in Fiber Direction. 10.1.6 Interlaminar Damage and Delamination of Laminate Composites. 10.1.7 Damage Mesomodel of Laminates. 10.2 Elastic-Brittle Damage of Ceramic Matrix Composites. 10.2.1 Elastic-Brittle Damage Behavior of Ceramic Matrix Composites and Its Unilateral Effect. 10.2.2 Damage Variable and Thermodynamic Potential. 10.2.3 Damage Potential and Evolution Equation of Damage. 10.2.4 Application to Ceramic Matrix Composites. 10.3 Local Theory of Metal Matrix Composites. 10.3.1 Local and Overall Configurations of Composites. 10.3.2 Local-Overall Relations of Stress and Strain. 10.3.3 Strain- and Stress-Concentration Factors in Matrixand Fiber. 10.3.4 Local-Overall Relations for Damage State. 10.3.5 Stress- and Strain-Concentration Factors in Fictitious Undamaged and Current Damaged Configurations. 10.3.6 Local Stress in Uniaxial Tension of Composites

11 Local Approach to Damage and Fracture Analysis

11.1 Local Approach to Fracture Based on Continuum Damage Mechanics and Finite Element Method. 11.1.1 Local Approach to Fracture, Modeling of Fracture. 11.1.2 Mesh-Sensitivity in Local Approach to Fracture. 11.2 Mesh-Sensitivity in Time-Independent Deformation. 11.2.1 Stain-Softening and Mesh-Sensitivity. 11.2.2 Bifurcation of Deformation and Strain Localization. 11.3 Regularization of Strain and Damage Localization in Time-Independent Materials. 11.3.1 Limitation of MeshSize. 11.3.2 Mesh-Dependent Softening Modulus – Modification of Material Property by Mesh-Size. 11.3.3 Nonlocal Damage Theory. 11.3.4 Gradient-Dependent Theory. 11.3.5 Cosserat Continuum. 11.3.6 Artificial Viscosity. 11.4 Mesh-Sensitivity in Time-Dependent Deformation. 11.4.1 Mesh-Sensitivity in Creep Crack Analysis. 11.4.2 Bifurcation and Localization in Time- Dependent Deformation. 11.5 Causes of Mesh-Sensitivity in Time-Dependent Deformation. 11.5.1 Stress Singularity at Crack Tip. 11.5.2 Stress-Sensitivity of Damage Evolution Equation and Damage Localization. 11.5.3 Regularization of Mesh-Sensitivity in Time-Dependent Deformation. 11.5.4 Analysis of Creep-Crack Extension by Means of Nonlocal Damage Theory

Recommended Literature

1. Kachanov LM. Foundations of fracture mechanics (in Russian). Izdatielistvo Nauka, Moscow, 1974.
2. Kachanov LM (1986) Introduction to continuum damage mechanics. Martinus Nijhoff, Dordrecht
3. Lemaitre J (1992) A course on damage mechanics. Springer, Berlin; 2nd Edition (1996)
4. Lemaitre J, Desmorat R (2005) Engineering damage mechanics. Springer, Belrin
5. Krajcinovic D (1996) Damage mechanics. North-Holland, Amsterdam
6. Kattan PI, Voyiadjis GZ (2002) Damage mechanics with finite elements. Springer, Berlin
7. Voyiadjis GZ, Kattan PI (1999) Advances in damage mechanics: metals and metal matrix composites. Elsevier, Amsterdam
8. Stepanova L.V. *Mathematical Methods of Fracture Mechanics*. Moscow: Fizmatlit, 2009.
9. Zehnder A.T. Fracture Mechanics. Springer London Dordrecht Heidelberg New York : Springer, 2012. 223 p.
10. J.R. Rice, in *Fracture an Advanced Treatise*, vol. 2, ed. by H. Liebowitz (Academic Press, New York, 1968), pp. 191–311, chap. 3
11. Lawn B. *Fracture of Brittle Solids*, 2nd edn. Cambridge University Press: Cambridge, 1993.
12. Suresh S. *Fatigue of Materials*, 2nd edn. Cambridge University Press: Cambridge 1998.
12. M. Janssen, J. Zuidema, R.Wanhill, *Fracture Mechanics*, 2nd edn. London: Spon Press, 2004.
14. T.L. Anderson, *Fracture Mechanics Fundamentals and Applications*, 2nd edn. Boca Raton: CRC Press, 1995.
15. Sanford R.J. *Principles of Fracture Mechanics* New York : Prentice Hall, 2003.
16. Hellan K. *Introduction to Fracture Mechanics* New York: McGraw-Hill, 1984.
17. Broberg K.B. *Cracks and Fracture* San Diego: Academic Press, 1999.
18. Murakami Y. Metal *Fatigue: Effects of Small Defects and Nonmetallic Inclusions*. Oxford: Elsevier, 2002. 390 p.

19. Murakami S. *Continuum Damage Mechanics A Continuum Mechanics Approach to the Analysis of Damage and Fracture*. Springer, Dordrecht, Heidelberg, London, New York: Springer, 2012. 423 p.
20. Krupp U. *Fatigue Crack propagation in metals and alloys. Microstructural Aspects and Modelling Concepts*. Weinheim: Willey, 2007. 314 p.
21. Pestrikov V.M., Morozov V.M. Fracture Mechanics.Lecture Course. St. Petersburg: Profession, 2012. 552 p.
22. Liu A.F. Mechanics and Mechanisms of Fracture: An Introduction. Ohio: ASM International Materials Park, 2005. 467 p.

Exercises. Consider the plate shown in Figure and simulate the crack growth process accounting for the damage accumulation process.